DOCUMENT RESUME

ED 281 758 SE 047 952

AUTHOR Lester, Frank K., Jr.; Garofalo, Joe
TITLE The Influence of Affects, Beliefs, and Metacognition on Problem Solving Behavior: Some Tentative Speculations.
PUB DATE Apr 87
NOTE 24p.; Paper presented at the Annual Meeting of the American Educational Research Association, (Washington, DC, April 20-24, 1987).
PUB TYPE Reports - Research/Technical (143) -- Speeches/Conference Papers (150) -- Viewpoints (120)

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS Educational Research; *Grade 7; Interviews; *Mathematics Instruction; *Metacognition; *Problem Solving; Secondary Education; *Secondary School Mathematics; Socioeconomic Influences; *Student Attitudes
IDENTIFIERS *Mathematics Education Research

ABSTRACT

Some ideas are presented about the ways in which the three domains of metacognition, affective factors, and beliefs influence the cognitive activities of seventh graders as they solve mathematics problems. The research project, initiated in 1981, is first described. Theoretical considerations are then discussed, with sections on affects, beliefs, control, sociocultural conditions, and the relationship among these domains and the domain of knowledge. Observations of seventh graders are presented, with a preliminary conceptualization of the role of perseverance. Problem-solving scenarios are provided, with contrasts between the responses of an average and an above-average student for the first scenario and reactions of other students for the other three scenarios. In the concluding discussion, the researchers note that their observations to date are speculative and likely to be altered as they continue their investigations. (MNS)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED281758

THE INFLUENCE OF AFFECTS, BELIEFS, AND METACOGNITION ON PROBLEM SOLVING BEHAVIOR: SOME TENTATIVE SPECULATIONS

Frank K. Lester, Jr.

Indiana University

Joe Garofalo

State University of New York at Buffalo

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

☒ This document has been reproduced as received from the person or organization originating it.

☐ Minor changes have been made to improve reproduction quality.

• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY

Frank K Lester, Jr.

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

SE 047 952

Paper prepared for the annual meeting of the American Educational Research Association, Washington, D.C., April, 1987. Research reported in this paper is supported by National Science Foundation Grant # MDR 85-50346. Any opinions, conclusions, or recommendations expressed are those of the authors and do not necessarily reflect the views of the National Science Foundation.

THE INFLUENCE OF AFFECTS, BELIEFS, AND METACOGNITION ON PROBLEM SOLVING BEHAVIOR: SOME TENTATIVE SPECULATIONS

Frank K. Lester, Jr.
Indiana University
Joe Garofalo
State University of New York at Buffalo

Any good mathematics teacher would be quick to point out that students' success or failure in solving a problem often is as much a matter of self-confidence, motivation, perseverance, and many other non-cognitive traits, as of what mathematical knowledge they possess. Nevertheless, it is safe to say that the overwhelming majority of problem-solving researchers have been content to restrict their investigations exclusively to cognitive aspects of performance. Such a restricted posture may be natural for psychologists and artificial intelligence scientists who are concerned primarily with expert systems or machine intelligence, but it simply will not suffice for the study of problem solving in school contexts (McLeod, 1985; Schoenfeld, 1985).

In January, 1986 we began a study of the role of metacognition[1] in the mathematical problem-solving behavior of seventh

---

[1] While metacognition is an underresearched area, quite a lot has been written about it in recent years. The reader who wishes to learn more about what metacognition involves and the role it plays in mathematical activity should refer to Garofalo & Lester (1985) and Schoenfeld (1985).

grade students. This research effort follows about five years of preliminary work in this area by us and, while we are convinced that metacognition plays a vital role in problem solving, we regard it as but one of a number of "driving forces" (to use the term coined by Silver [1982] and elaborated on by Schoenfeld [1985]). In particular, two other domains seem especially important: affective factors and beliefs.

In this paper we present some ideas about the ways in which these three domains influence the cognitive activities of seventh graders as they solve mathematics problems. It is perhaps important to add that since this paper is being written while in the midst of our research, our views are largely speculative and readily subject to change. As we get further into the analysis of the overwhelming amount of data that has been (and is being) collected we are likely to alter our views dramatically.

Description of Our Research Project

In 1981 we initiated an exploratory investigation of young children's metacognitive awareness as it relates to mathematical performance which ultimately led to the development of a framework which specifies key points where metacognitive actions are likely to affect cognition (Garofalo & Lester, 1985; Lester & Garofalo, 1982). The original objectives of our present research were the following: (1) to assess the metacognitive behaviors of grade seven children and to investigate the role of metacognition in their mathematical problem-solving activity, and (2) to ex-

plore the extent to which these students can be taught to be more self-aware of their problem-solving behaviors and to monitor and evaluate these behaviors. The scope has been expanded to include consideration of certain affective factors as well as students' beliefs about mathematics and problem solving. More specifically, we are now also interested in factors that affect students' self-confidence in solving problems, the conditions under which they persevere in problem-solving efforts, and how student beliefs influence problem-solving behavior. Reasons for the choice of these particular factors are discussed in the section, Theoretical Considerations.

The project consists of four phases: I. preparation of suitable mathematical tasks; II. pre-treatment assessment of mathematical performance and of metacognitive behaviors; III. instructional treatment; and IV. post-treatment assessment. (phases I through III will have been completed by the date of presentation of this paper.)

Both the pre- and post-treatment assessments involve performance on written tests of routine and non-routine mathematics problems, performance of selected students on routine and non-routine problems observed through individual interviews, and performance of selected students observed as they worked in pairs on non-routine problems. The individual and pair interviews were both audio- and video-taped and performance is now being analyzed in terms of behaviors exhibited, strategies used, and decisions

made. In addition, students were asked to view the tapes of their own performance and to respond to questions concerning their activities. These responses are also being analyzed.

The instructional treatment was presented two or three days per week for a period of 12 weeks and consisted of three metacognitive components: the teacher as external monitor, the teacher as facilitator of students' metacognitive development, and the teacher as model. Nearly all of the instructional sessions were audio- and video-taped for later analysis.

The subjects for this study are students in two seventh-grade mathematics classes in Bloomington, Indiana. One class is a regular level class, the other is an advanced-level class. Both classes are taught regularly by the same teacher and the instructional treatment was taught to both classes by Frank Lester, who is the project director. More detail about each phase of the project is given in an earlier report (Lester & Garofalo, 1986).

As is evident from the preceding description, investigating the role of metacognition and the extent to which these students can be made to be more metacognitively aware are the primary foci of our research. However, for reasons described in the next section, selected affects and beliefs are also being considered. In the next section we discuss some of the theoretical considerations involved in determining how these factors fit into the framework of the study.

Theoretical Considerations

We begin by postulating that an individual's failure to solve a problem successfully when the individual possesses the necessary knowledge[2] stems from the presence of non-cognitive and metacognitive factors that inhibit the appropriate utilization of one or more pieces of necessary knowledge. These factors are of four types: affects, beliefs, control, and socio-cultural conditions. In the following sections we discuss each of these types.

Affects

This domain involves individual feelings and includes attitudes and emotions. Problem-solving research involving attitudes often has been limited to examinations of the correlation between attitudes and problem-solving performance (Silver, 1985). Not surprisingly, attitudes that have been shown to be related to problem solving include: motivation, interest, confidence, perseverance, and willingness to take risks (Lester, 1980). In addition to the foregoing, Webb, Moses, & Kerr (1977) have suggested that tolerance of ambiguity and resistance to premature closure are also important correlates of problem-solving performance. Unfortunately, as McLeod (1985) has pointed out, the

---

[2] "Knowledge" refers to both formal and informal mathematical knowledge, knowledge of heuristics (e.g., drawing pictures, looking for patterns, working backwards, devising suitable notation), and knowledge of contextual information present in a problem statement (e.g., knowledge that a quarter is worth 25 cents in a problem involving money).

theoretical underpinnings of the research in this area have been very weak.

Typically, emotions are subjective reactions to specific situations. Of course, emotions can have either a facilitating or debilitating effect on the individual, but negative emotions (e.g., frustration) are not necessarily debilitating nor are positive emotions (e.g., joy) necessarily facilitating. There is a growing body of research to support the notion that emotions and cognitive actions interact in important ways (Mandler, 1986). Most of this research has been restricted to the study of the conditions under which certain emotions occur or to the nature of individuals' behavior when in a particular emotional state.

We hold the position that attitudes are traits, albeit perhaps transient ones, of the individual, whereas emotions are situation-specific states. An individual may have developed a particular attitude toward a class of problems which affects her or his performance on a specific problem in that class (e.g., a student may greatly dislike problems involving percents). At the same time, a particular problem may give rise to an unanticipated emotion (e.g., frustration may set in when a student finds that he or she has made little progress toward solving a problem after working diligently on it for a considerable amount of time). At present, we are interested in two attitudes: <u>confidence</u> and <u>perseverance</u>. Confidence includes the individual's belief in her or his ability to succeed even in solving "hard problems" as well as

self-confidence with respect to her or his peers. Perseverance includes three dimensions: desire to obtain correct answers, resistance to premature closure, and persistence[3].

Beliefs

Schoenfeld (1985) refers to beliefs, or "belief systems" to use his term, as the individual´s mathematical world view; that is, "...the perspective with which one approaches mathematics and mathematical tasks" (Schoenfeld, 1985, p. 45). Beliefs constitute the individual´s subjective knowledge about self, mathematics, problem solving, and the topics dealt with in problem statements. An individual´s beliefs may or may not be valid as items of true knowledge. Beliefs can be distinguished from knowledge in two ways: a belief may not be true, whereas knowledge must be true; and a belief may not be well-justified, whereas knowledge must be (cf., Kitcher, 1984). It is clear to us that beliefs shape attitudes and emotions and direct the decisions made during problem solving. In our research we are particularly interested in the students´ beliefs about their own capabilities and limitations (beliefs about self) and about the nature of problem solving.

---

[3] Desire to obtain the right answer refers to the attitude of sticking with a task until the correct answer is found. Premature closure refers to the desire to obtain an answer, any answer, in order to be finished with a task. Persistence refers to the attitude of not giving up too quickly, of staying on task without regard for getting a correct answer.

Control

We use the term "control" in much the same way as Schoenfeld (1985). That is, control refers to the marshalling and subsequent allocation of available resources to solve a problem. More specifically, it deals with executive decisions about planning, evaluating, monitoring, and regulating. Two aspects of control processes are of particular interest to us: knowledge about cognition and regulation of cognition -- that is, metacognition. We will not discuss metacognition here since information about our work in this area is available elsewhere (Garofalo & Lester, 1985; Lester & Garofalo, 1986).

Socio-cultural Conditions

In recent years, the point has been raised within the cognitive psychology community that human intellectual behavior must be studied in the context in which it takes place (Neisser, 1976; Norman, 1981). That is to say, since human beings are immersed in a reality that both affects and is affected by human behavior, it is essential to consider the ways in which socio-cultural factors influence cognition. In the case of mathematical behavior, D´Ambrosio (1985) argues that children bring to school their own mathematics which has developed within their own socio-cultural environment. This mathematics, called "ethnomathematics" by D´Ambrosio, provides the individual with a wealth of intuitions and informal procedures for dealing with mathematical phenomena

(cf., Carraher, Carraher & Schliemann, 1985; Rogoff & Lave, 1984). The point then is that the wealth of socio-cultural conditions which make up an individual's reality play a prominent role in determining the individual's potential for success in solving a given problem. Indeed, it is our view that these conditions have a direct influence on each of the other domains.

Relationship Among the Domains

Figure 1 depicts the four domains discussed in the previous paragraphs. The arrows in the figure indicate our present perspective about the relationships among the domains. Thus, socio-cultural conditions directly influence the formation of attitudes, emotions, and beliefs, as well as control and knowledge utilization. Affects influence both control and knowledge but not beliefs or socio-cultural conditions, whereas beliefs directly influence all the other domains except socio-cultural conditions and control directs only the ways in which knowledge is utilized.

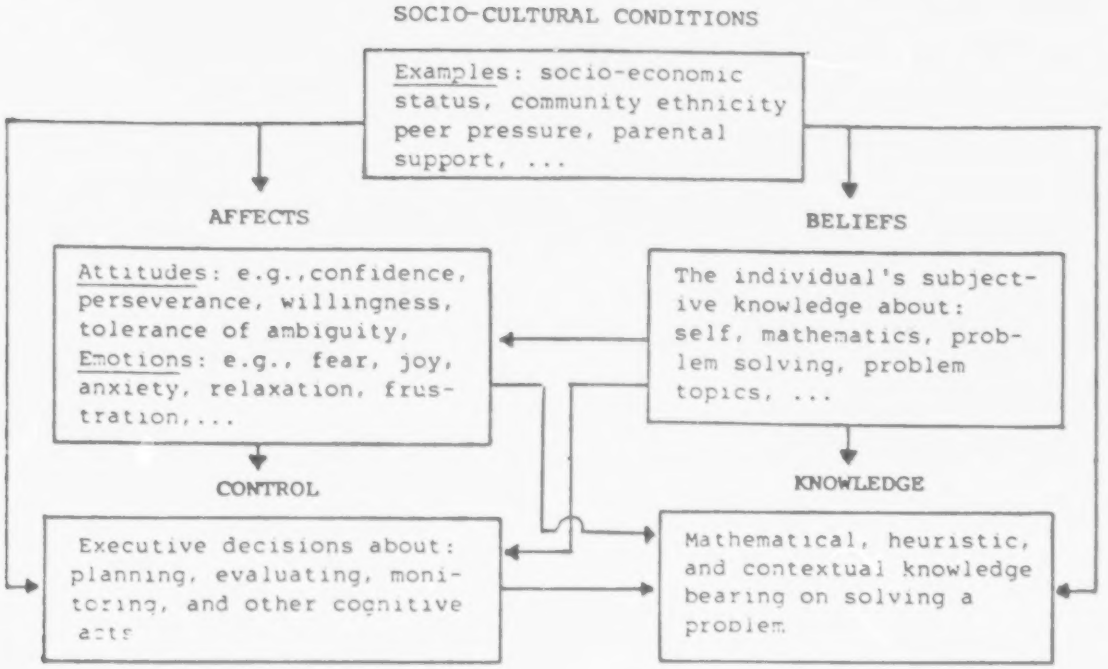

SOCIO-CULTURAL CONDITIONS
Examples: socio-economic status, community ethnicity peer pressure, parental support, ...
AFFECTS
BELIEFS
Attitudes: e.g.,confidence, perseverance, willingness, tolerance of ambiguity, Emotions: e.g., fear, joy, anxiety, relaxation, frustration,...
The individual's subjective knowledge about: self, mathematics, problem solving, problem topics, ...
CONTROL
KNOWLEDGE
Executive decisions about: planning, evaluating, monitoring, and other cognitive acts
Mathematical, heuristic, and contextual knowledge bearing on solving a problem

Figure 1: Relationship among types of non-cognitive and metacognitive factors that influence knowledge utilization in problem solving

Observations of Seventh Graders

To illustrate how we are currently proceeding in our consideration of affects and beliefs, we provide a preliminary conceptualization of the role of perseverance and we present scenarios adapted from our current work with seventh graders. The scenarios serve to point out the, sometimes dominant, influence non-cognitive factors can have on problem-solving performance. These scenarios are condensed accounts of problem-solving episodes during either one-on-one interviews or classroom instruction.

Conditions of Perseverance

Our decision to look at the role of perseverance in problem solving necessitated a consideration of the conditions under which it occurs. Tentatively, we have identified three key conditions under which to investigate the presence or absence of perseverance: (I) I know how to do it but it is tedious or boring; (II) Maybe I can get it, I'm not sure about part of it; (III) I have no idea as to how to proceed. Each of these three conditions can give rise to various personal reactions. For example, under condition I an individual may become bored due to the tedious nature of the work necessary to solve a problem; under condition II a problem solver may feel very unsure about her or his chances to solve a problem and proceeds rather cautiously as a result; and under condition III the feeling of being "lost at sea" may cause frustration to set in. Of course, these conditions not only are dependent upon the problem solver but they interact with the type of task involved. That is, some problems have more potential for establishing one set of conditions than another and the conditions can change as a result of actions taken by the problem solver during work on a problem. Thus, the researcher interested in perseverance would want to select problems that have potential for establishing each of the three sets of conditions.

Some Problem-solving Scenarios

Scenario I: During an individual problem-solving session that took place prior to the beginning of the instructional treatment, two students attempted to solve the following problem:

> Atlas Steel makes 4 different types of steel. From a shipment of 300 tons of raw steel, the factory produced 60 tons of type I, which sold for $60 a ton; 75 tons of Type II, which sold for $65 a ton; 120 tons of Type III, which sold for $72 a ton; and 45 tons of Type IV, which sold for $95 a ton. Raw steel costs $40 a ton. It costs the factory $2500 to convert every 300 tons of raw steel into the 4 types. How much profit did Atlas Steel make on this shipment?

Contrasts Between an Average and an Above Average Student

In response to a series of queries from the interviewer, WC, an average ability student, made the following comments: "I like this kind of problem 'cause it makes me think and it's challenging." "I love math; its not like the other classes where you either know it or you don't." While working on the Atlas Steel problem, WC seemed to be very confused but she did not become flustered until after quite a long time. She persisted even though she had a strong notion that she wasn't getting anywhere. Her general approach is to view all problems as "routine" problems and to use as a "fall back" strategy: "When in doubt, try all operations one at a time." For example, on the Atlas Steel problem first she added weights, then costs, then gave up on addition. Next she began to multiply costs by weights. After she added up the resulting products she didn't know what to do, so she began dividing the products by 300. At no point did she seem

confident in her actions. Despite the large amount of tedious calculations, she still felt challenged by the problem and so persisted. Furthermore, the only monitoring of her activities was to check the results of her calculations. If they checked, she did no further evaluation. WC's belief that problem solving is simply a matter of trying all four operations and then picking the one that "makes the most sense," overrides any need to monitor the appropriateness of her actions.

By contrast, AA, one of the best students in the advanced class, devoted a considerable amount of time to "...figuring out what to do" (her words). She said: "Confidence depends on feeling that I understand how to do it, not in getting the right answer." And, "you're proud of yourself when you get [solve] a weird problem." AA seemed to be a bit embarrassed at first because she had to "...just sit here and think about the problem," an unusual situation for her apparently. Another important difference between AA and WC is that AA did not want to check her calculations on the Atlas Steel problem because "its boring." She checked primarily by estimating. She said that sometimes she does not need to check her calculations at the end because "it takes too much time and I'm usually pretty sure that it's right."

This scenario illustrates some very important non-cognitive differences between average and above average problem solvers. In spite of having a very positive attitude toward mathematics, WC's progress is hampered greatly by a strongly held belief and

by a consequent lack of metacognitive behavior. She has developed the belief that all mathematics problems can be solved by simply performing a series of computations. She sees no need to worry about making sense of the problem. Furthermore, in WC's behavior we see an example of debilitating perseverance, in this case negative persistence. She was willing to continue to work on the problem even though she did not have any sense that what she was doing would get her anywhere.

The other student, AA, also enjoys mathematics very much and she has developed considerable confidence in her ability to tackle successfully difficult problems. This confidence may stem from a natural reflectiveness that has proven useful to her in the past to solve "weird" problems. Also, unlike WC, she exhibited on-going monitoring and regulating behavior. It is possible that this metacognitive behavior is a result of her dislike for the tedium she associates with checking messy calculations.

Scenario II: The students in an average-ability class were asked by the teacher to solve the following problem individually without consulting with their classmates:

> A pair of day-old mice escaped from their pen and got loose in the basement. It takes mice 1 month to mature and another month to produce a new pair of mice each month. The new pairs of mice grow and reproduce at the same rate as the original pair and each pair has a male and a female. If no mice die, how many pairs of mice will there be after 8 months?

While walking around the classroom as the students worked on the problem, the teacher had the following exchanges with two students, AW and KJ:

Exchange A

AW: Who cares about solving problems about some dumb mice?

T: You are pretty hard to please. What is it about this problem that you don't like?

AW: It's boring! Why can't these problems be about something interesting once in a while?

T: I'll make a deal with you. If you make a list of things that interest you, I'll try to make up some problems using your ideas. But for now I want you to give this one a try.

AW: Well OK (unenthusiastically), maybe I will.

.

.

.

Exchange B

KJ: Is this right? I got 128 mice.

T: Did you check what you did carefully?

KJ: Yeah! Look, it's easy. You just double. There's nothing else to check.

T: Well, maybe you are overlooking some important information that might change your answer.

KJ: I think it's right. Why don't you just tell me?

.

.

.

In Exchange A of the second scenario, knowing that AW exerts very little effort to solve any of the problems the teacher gives to the class despite above grade level performance on standardized mathematics achievement tests, suggests that his failure is due to something other than a lack of knowledge. AW actually does quite well when he tries. Unfortunately, he often does not try at all. To say that he is unmotivated is putting it mildly. It is quite common for AW simply to write down an answer to a problem in order to be finished as quickly as possible (lack of resistance to premature closure -- lack of perseverance, type 2).

A different obstacle is hindering KJ's success. He has a history of wanting to get answers to problems quickly and moving on to something else. Like AW, he tends to seek premature closure on tasks and wants to be immediately reinforced by the teacher for his answers. Furthermore, he believes that "checking" is nothing more than looking over your calculations (belief about problem-solving processes).

Scenario III: The class was given the following problem to solve cooperatively in groups of four:

> Carla is the drummer in a band. On Tuesday she received her paycheck for playing with the band for one month. She spent 20% of it that day and 50% of what was left on Wednesday. She then had $50 left. How much did Carla receive in her paycheck?

The teacher notices that one group is not working on the problem. This exchange takes place:

T: What's wrong, does your group have a question?

EN: We can't do it. We don't know how.

T: Sure you can. Do all of you feel the same way?

(all 4 in unison: Yeah.)

T: You say you don't know how. Is there something you don't understand about the problem?

EN: Yes, we can't do percents.

T: Come on now. You're kidding me. You worked harder percent problems than this in the fifth grade.

JE: But not this year. We haven't done percent yet this year.

.

.

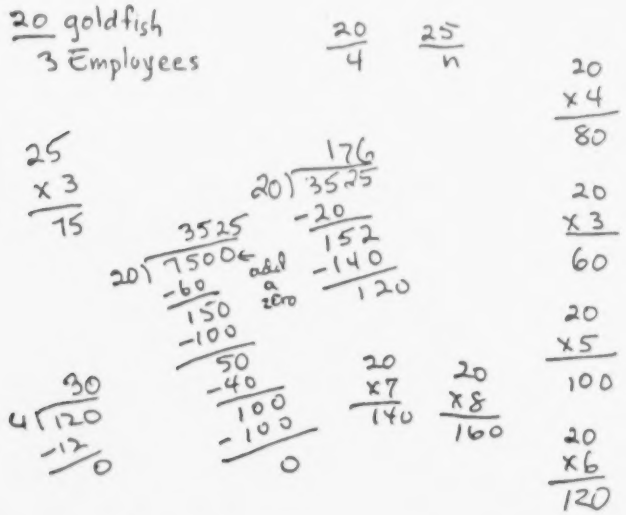
.

This scenario illustrates a continuing source of frustration for the teacher. Each of the four students in this group has at least average mathematics ability and each is attentive and cooperative in class. The teacher knows that they have the necessary content knowledge to solve this problem, or at least they know enough to get started. However, it appears that they do not believe they can do it. They have convinced themselves that percent problems are too difficult for them and, consequent-

ly, it will do no good to even try (belief about a mathematics topic, lack of self-confidence).

Scenario IV: Below is the work shown by one student on a problem given as an in-class assignment. Students were expected to work alone and they had about 20 minutes to solve it. The student is somewhat below average in mathematics achievement.

> Problem: One really cold winter night the furnace failed at the "Tropicana Fish Shop". Because of this the temperature dropped in the shop and 20 goldfish died. To replace the fish the shop manager bought as many goldfish as he had left. Then as a precaution he divided all the goldfish equally among himself and 3 employees and each person took his share home. The manager got 25 goldfish. How many goldfish were in the shop before the night the furnace failed?



Your answer: 176 Fish in all

This final scenario points, among other things, to an absence of self-regulation, an aspect of metacognition. The student was apparently unconcerned about the appropriateness of her actions and she seemed perfectly willing to continue to perform any number of calculations as long as the results of her efforts

were correct. At no point is there any evidence that she may have asked herself: "Do I understand this problem. Is what I am doing getting me anywhere?" Furthermore, when she finally arrived at an answer, she did not seem to be concerned about its unreasonableness.

Discussion

In an earlier section of this paper we suggested that success in solving mathematics problems involves much more than the possession of adequate knowledge. In particular, we posited that various non-cognitive and metacognitive factors have tremendous influence on problem-solving activity: viz., affects, beliefs, control processes, and socio-cultural conditions. Unfortunately, in the absence of an adequate theoretical framework to guide us, our work has been somewhat haphazard. For this reason we hasten to emphasize that our observations to date are extremely speculative and likely to be altered substantially as we continue our investigations.

References

Carraher, T. N., Carraher, D. W., & Schliemann, A.D. (1985). Mathematics in the streets and in the schools. British Journal of Developmental Psychology, 3, 21-29.

D´Ambrosio, U. (1985). Da realidade a acão: Reflexões sobre educação e matemática. São Paulo, Brazil: Summus Editorial.

Garofalo, J. & Lester, F. K. (1985). Metacognition, cognitive monitoring and mathematical performance. Journal for Research in Mathematics Education, 16, 163-176.

Kitcher, P. (1984). The nature of mathematical knowledge. New York: Oxford University Press.

Lester, F. K. (1980). Research on mathematical problem solving. In R. Shumway (Ed.), Research in mathematics education. Reston, VA: National Council of Teachers of Mathematics.

Lester, F. K. & Garofalo, J. (1982, April). Metacognitive aspects of elementary school students´ performance on arithmetic tasks. Paper presented at the annual meeting of the American Educational Research Association, New York.

Lester, F. K. & Garofalo, J. (1986, April). An emerging study of sixth graders´ metacognition and mathematical performance. Paper presented at the annual meeting of the American Educational Research Association, San Francisco.

Mandler, G. (1986, May). Affect and learning: Causes and consequences of emotional interactions. Paper prepared for the workshop on "Affective Influences on Cognitive Processes," San

Diego.

McLeod, D. B. (1985). Affective issues in research on teaching mathematical problem solving. In E. A. Silver (Ed.), Teaching and learning mathematical problem solving: Multiple research perspectives. Hillsdale, NJ: LEA.

Neisser, U. (1976). General, academic, and artificial intelligence. In L. Resnick (Ed.), The nature of intelligence. Hillsdale, NJ: LEA.

Norman, D. A. (1981). Twelve issues for cognitive science. In D. A. Norman (Ed.), Perspectives in cognitive science. Norwood, NJ: Ablex.

Rogoff, B. & Lave, J. (Eds.), (1984). Everyday cognition: Its development in social context. Cambridge, MA: Harvard University Press.

Schoenfeld, A. H. (1985). Mathematical problem solving. Orlando, FL: Academic Press.

Silver, E. A. (1982, January). Thinking about problem solving: Toward an understanding of metacognitive aspects of mathe matical problem solving. Paper prepared for the Conference on Thinking, Fiji.

Silver, E. A. (1985). Research on teaching mathematical problem solving: Some underrepresented themes and needed directions. In E. A. Silver (Ed.), Teaching and learning mathematical problem solving: Multiple research perspectives. Hillsdale, NJ: LEA.

Webb, N. L., Moses, B. E., & Kerr, D. R. (1977). Developmental activities related to summative evaluation (1975-1976). Technical Report IV of the Final Report of the Mathematical Problem Solving Project. Bloomington, IN: Mathematics Education Development Center, Indiana University.